B Solutions to Problems

Solution to Problem 1.

- Stages:

$$t \leftrightarrow \begin{cases} \text{production run } t & \text{if } t = 1, 2, 3 \\ \text{end of process} & \text{if } t = 4 \end{cases}$$

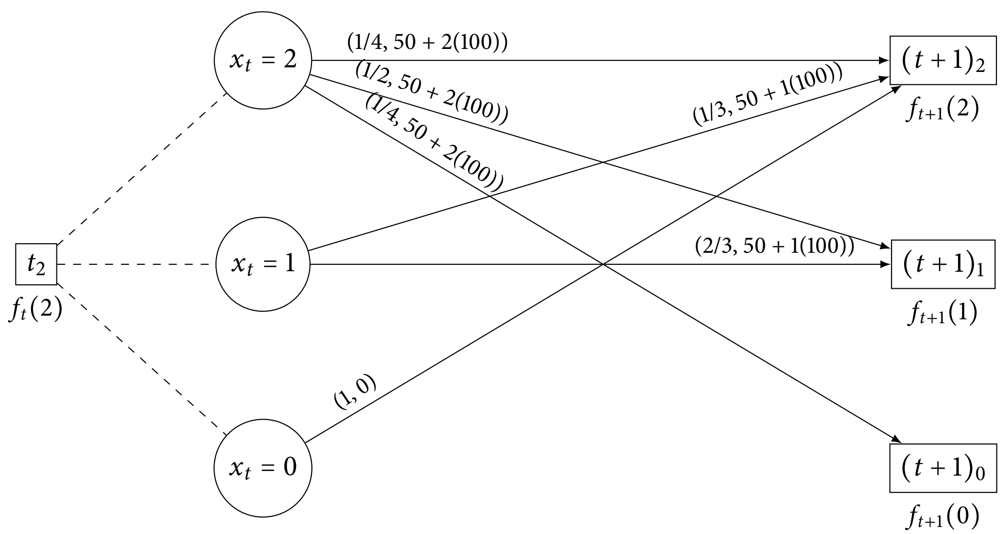
- States:

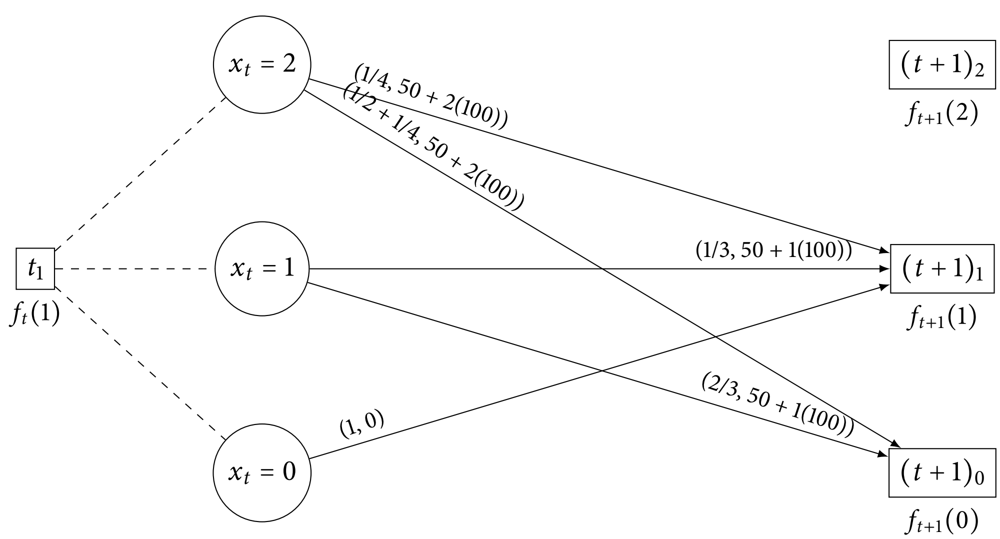
$$n \leftrightarrow \text{number of acceptable missiles left to produce} \quad \text{for } n = 0, 1, 2$$

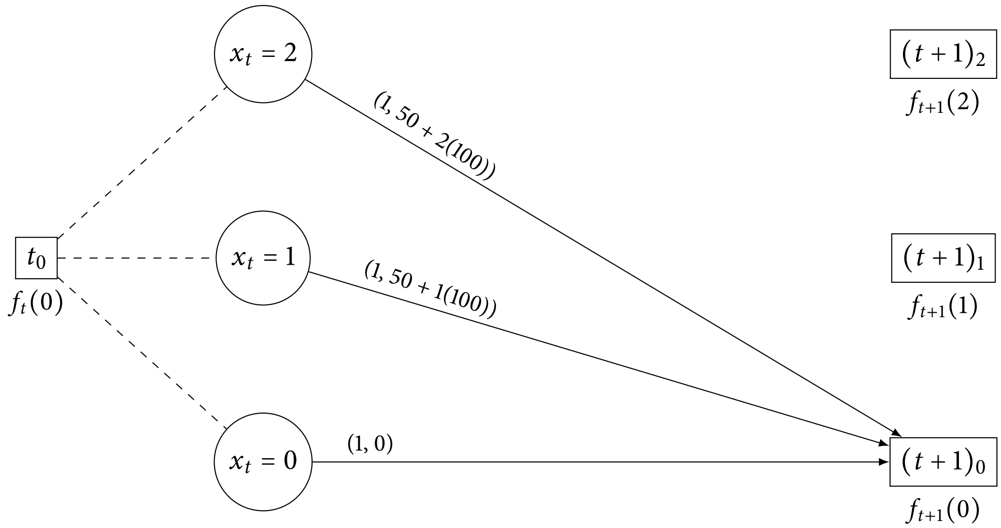
- Allowable decisions x_t at stage t and state n:
  - Let x_t represent the number of missiles to produce in production run t
  - At stages $t = 1, 2, 3$ and states $n = 0, 1, 2$, x_t must satisfy:

$$x_t \in \{0, 1, 2\}$$

- Basic structure:
  - I've included a sketch for every state/decision combination, so you can explicitly see what's going on.
  - Edge labels represent (transition probability, contribution)







- In words, the value-to-go $f_t(n)$ at stage t and state n is the minimum total expected cost for production runs $t, t+1, \dots, 3$ if we start with n missiles remaining to produce

- Value-to-go recursion:

$$f_t(2) = \min \Bigg\{ \underbrace{f_{t+1}(2)}_{x_t=0}, \underbrace{\frac{1}{3}\big(50 + 1(100) + f_{t+1}(2)\big) + \frac{2}{3}\big(50 + 1(100) + f_{t+1}(1)\big)}_{x_t=1},$$
$$\underbrace{\frac{1}{4}\big(50 + 2(100) + f_{t+1}(2)\big) + \frac{1}{2}\big(50 + 2(100) + f_{t+1}(1)\big) + \frac{1}{4}\big(50 + 2(100) + f_{t+1}(0)\big)}_{x_t=2} \Bigg\}$$

$$f_t(1) = \min \Bigg\{ \underbrace{f_{t+1}(1)}_{x_t=0}, \underbrace{\frac{1}{3}\big(50 + 1(100) + f_{t+1}(1)\big) + \frac{2}{3}\big(50 + 1(100) + f_{t+1}(0)\big)}_{x_t=1},$$
$$\underbrace{\frac{1}{4}\big(50 + 2(100) + f_{t+1}(1)\big) + \Big(\frac{1}{2} + \frac{1}{4}\Big)\big(50 + 2(100) + f_{t+1}(0)\big)}_{x_t=2} \Bigg\}$$

$$f_t(0) = \min \Bigg\{ \underbrace{f_{t+1}(0)}_{x_t=0}, \underbrace{50 + 1(100) + f_{t+1}(0)}_{x_t=1}, \underbrace{50 + 2(100) + f_{t+1}(0)}_{x_t=2} \Bigg\}$$

- Boundary conditions:

$$f_4(2) = 25$$
$$f_4(1) = 25$$
$$f_4(0) = 0$$

- Desired value-to-go function value: $f_1(2)$

Solution to Problem 2.

- Stages:

$$t \leftrightarrow \begin{cases} \text{beginning of year } t & \text{if } t = 1, 2, 3 \\ \text{end of process} & \text{if } t = 4 \end{cases}$$

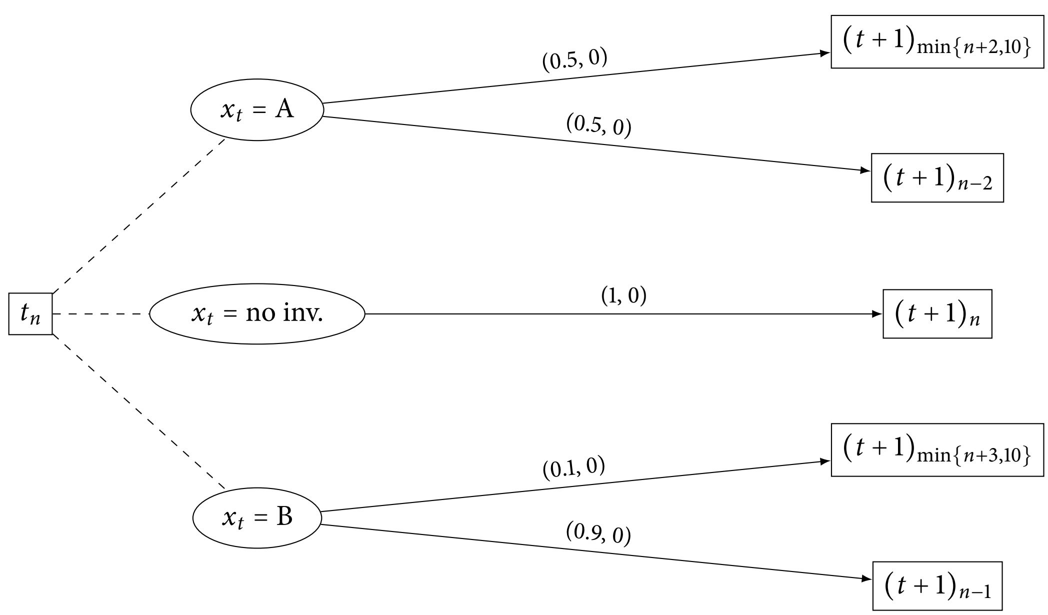
- States:

$$n \leftrightarrow \text{amount of money in account (in \$ millions)} \quad \text{for } n = 0, 1, \dots, 10$$
$$(n = 10 \text{ corresponds to having \$10 million or greater in the account})$$

- Allowable decisions x_t at stage t and state n:
  - Let x_t represent the investment chosen in year t
  - At stages $t = 1, 2, 3$ and states $n = 0, 1, \dots, 10$, x_t must satisfy:

$$\begin{aligned} x_t &\in \{\text{A}, \text{B}, \text{no investment}\} && \text{for } n = 5, \dots, 10 \\ x_t &\in \{\text{A}, \text{no investment}\} && \text{for } n = 3, 4 \\ x_t &\in \{\text{no investment}\} && \text{for } n = 0, 1, 2 \end{aligned}$$

- Basic structure – transition probabilities and contributions:
  - Arc labels represent (probability, contribution)



- In words, the value-to-go $f_t(n)$ at stage t and state n is the maximum probability of having at least \$10 million at the end of year 3, with years $t, t+1, \dots, 3$ remaining and starting with n million dollars to invest

- Value-to-go recursion: for $t = 1, 2, 3$,

$$f_t(n) = \max \left\{ \underbrace{0.5 f_{t+1}(\min\{n+2, 10\}) + 0.5 f_{t+1}(n-2)}_{x_t = \text{A}}, \right.$$

$$\underbrace{0.1 f_{t+1}(\min\{n+3, 10\}) + 0.9 f_{t+1}(n-1)}_{x_t=\text{B}}, \underbrace{1 f_{t+1}(n)}_{x_t=\text{no inv.}} \Bigg\} \quad \text{for } n = 5, \dots, 10$$

$$f_t(n) = \max \left\{ \underbrace{0.5 f_{t+1}(\min\{n+2, 10\}) + 0.5 f_{t+1}(n-2)}_{x_t=\text{A}}, \underbrace{1 f_{t+1}(n)}_{x_t=\text{no inv.}} \right\} \quad \text{for } n = 3, 4$$

$$f_t(n) = \max \left\{ \underbrace{1 f_{t+1}(n)}_{x_t=\text{no inv.}} \right\} \quad \text{for } n = 0, 1, 2$$

- Boundary conditions:

$$f_4(n) = 0 \quad \text{for } n = 0, 1, \dots, 9 \qquad f_4(10) = 1$$

- Desired value-to-go function value: $f_1(4)$